

Lorentz invariant ensembles of vector backgrounds

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Abstract

We consider gauge field theories in the presence of ensembles of vector backgrounds. While Lorentz invariance is explicitly broken in the presence of any single background, here, the Lorentz invariance of the theory is restored by averaging over a Lorentz invariant ensemble of backgrounds, i.e. a set of background vectors that is mapped onto itself under Lorentz transformations. This framework is used to study the effects of a non-trivial but Lorentz invariant vacuum structure or mass dimension two vector condensates by identifying the background with a shift of the gauge field. Up to now, the ensembles used in the literature comprise configurations corresponding to non-zero field tensors together with such with vanishing field strength. We find that even when constraining the ensembles to pure gauge configurations, the usual high-energy degrees of freedom are removed from the spectrum of asymptotic states in the presence of said backgrounds in euclidean and in Minkowski space. We establish this result not only for the propagators to all orders in the background and otherwise at tree level but for the full propagator.

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I. INTRODUCTION

Vector backgrounds appear in numerous sectors of physics. For example they can be used to include the influence of mass dimension two condensates into quantum chromodynamics (QCD) by shifting the gauge field and subsequently restoring Lorentz invariance by averaging over a Lorentz invariant ensemble of backgrounds [1, 2]. There this construction removes quarks and gluons from the spectrum of freely propagating particles. Lately, those condensates have attracted much attention in various respects (see e.g. [3]).

Analogous vector terms also occur in other domains. Introduced without the subsequent restoration of Lorentz invariance they are used in extensions of the standard model explicitly breaking Lorentz invariance [4]. The inclusion of such terms is motivated from string theory [5] and non-commutative field theory [6]. Lorentz invariance is one of the best-tested postulates of field theory but the current experimental status still leaves room for small deviations. However the approach under investigation in this article is Lorentz invariant.

We also identify the background vector with a shift of the gauge field. First however, in section II, we discuss the general framework for the inclusion of a dependence on an additional vector into a previously Lorentz invariant theory which *a priori* breaks Lorentz invariance explicitly but where the correlators are defined as the ensemble average over a Lorentz invariant set. Thereby a modified theory is obtained which preserves Lorentz invariance. A Lorentz invariant ensemble is a set of vectors which is mapped onto itself under any Lorentz trans-

formation, while, of course, almost every single element changes. As a next step, in section IIA, we carry out a classification of the weight functions which characterise those ensembles. At variance with euclidean space some subtleties arise in Minkowski space. Commonly the used sets contain configurations leading to vanishing and non-vanishing field tensors [1, 2]. Here, we limit the ensembles further by constraining them to pure gauge configurations. In this framework we analyse the objects central to the modified theory, i.e., the generating functional for the Green functions in section IIB and the fermionic two-point function by solving its equation of motion explicitly [7] in section IIC.

Finally, in section III, we summarise the paper, the main result being that the propagation of fermions over arbitrarily long distances is already stopped in ensembles of pure gauge configurations of the background for euclidean and Minkowski spaces characterised by their respective metrics. This result is not only derived to all orders in the background and otherwise at tree level but for the exact propagator.

Other observations are non-gaussianity of the resulting theory, structural similarities between the present approach and Lorentz invariant generalisations of chemical potentials as well as a technical relationship to stochastic field theory. Last but not least, in Minkowski space the modification of the fermionic two-point Green function amounts to a contribution of a scalar to the fermion's self energy but without external legs. This again indicates in a diagrammatic way that the ultraviolet degrees of freedom are removed from the asymptotic spectrum.

II. BREAKING AND RESTORING LORENTZ INVARIANCE

Regard a gauge field theory which is modified by including a dependence on a vector Φ . The translational invariance of the system remains intact, because the vector is constant. The Lorentz invariance of the theory is to be restored by taking the average over an ensemble of vectors Φ characterised by a Lorentz invariant weight $W(\Phi)$ [10]:

$$\langle \mathcal{O} \rangle_W = \int_{\Phi} W(\Phi) \mathcal{O}, \quad (1)$$

where \mathcal{O} stands for a generic operator, here and in the following, and with the normalisation condition:

$$\int_{\Phi} W(\Phi) = 1. \quad (2)$$

Apart from the case where $\Phi = 0$, which corresponds to the original theory, functions of Φ^2 are the only Lorentz invariant quantities that can be constructed from the vector Φ . The most general Lorentz invariant weight $W(\Phi)$ is given by the sum of an arbitrary normalisable function $w = w(\Phi^2)$ and a delta distribution $\delta^{(4)}(\Phi)$:

$$W(\Phi) = c\delta^{(4)}(\Phi) + w(\Phi^2) \quad (3)$$

In [1] the vector Φ represent a vector condensate translating the gauge boson field $A \rightarrow A + \Phi$. That system is investigated with a euclidean metric for quantum chromodynamics (QCD). In the sense $c = 0$ the weight chosen there ($\sim \exp\{-\Phi^2/\Lambda^2\}$) does not contain the unmodified theory. This manifests itself in the one-particle pole being removed from the quark and gluon propagators determined to all orders in Φ , meaning that there the partons do not propagate over arbitrarily long distances. They are no longer part of the asymptotic spectrum. The lowest order in an expansion for momenta large compared to the scale Λ^2 reproduces the standard free propagators.

In QCD the vector Φ also carries colour indices: $\Phi^2 = \Phi_{\mu}^a \Phi^{a\mu}$. The vector Φ is to transform homogeneously under gauge transformations whence any function of Φ^2 is gauge covariant. This also establishes the connection of the present approach with mass dimension two condensates because now Φ acts as a contribution to the gauge field [1, 2, 3]. Due to the non-abelian nature of the gauge theory, the ensemble of constant vectors characterised by a function of its square contains members which are pure gauge configurations and such leading to a non-vanishing field tensor. In our investigation, we will distinguish these cases by limiting the ensemble to vanishing field tensors. This in itself is a gauge invariant criterion. For the fermionic sector this leads to an analogue of quantum electrodynamics (QED) which we will study in the following. Further, we will compare the results for the different metrics.

A. Weight classification

Let us begin with a classification of the weight functions. In principle, in euclidean space the case $\Phi = 0$ is already included in $w(\Phi^2)$ as there $\Phi^2 = 0$ implies $\Phi = 0$. Nevertheless, in order to mark the potential contribution from the unmodified theory clearly, i.e., from $\Phi = 0$, let us split it off in form of a delta distribution in accordance with Eq. (3). The normalisation condition (2) then implies

$$\pi^2 \int_0^{+\infty} v \, dv \, w_E(v) = 1 - c \quad (4)$$

with $v := \Phi^2$ and where the subscript E marks the Euclidean case.

Every possible Lorentz invariant weight function $w_E(\Phi^2)$ can be reconstructed by a convolution with a delta weight

$$w_E(\Phi^2) = \int d\lambda \, \delta(\Phi^2 - \lambda) \, w_E(\lambda). \quad (5)$$

In this sense the delta weight:

$$w_E^{\{\lambda\}}(\Phi) := (4\pi\lambda)^{-1} \delta(\Phi^2 - \lambda) \quad (6)$$

can be seen as fundamental.

However, if, in the presence of a space with Minkowski metric, one wants to work in a time ordered formalism also in the theory with the background a different choice for the basis is better adapted. Noticing that:

$$2\pi i \delta(\Phi^2 - \lambda) = S_{\lambda}^{-}(\Phi) - S_{\lambda}^{+}(\Phi), \quad (7)$$

where

$$S_{\lambda}^{\pm}(\Phi) = (\Phi^2 - \lambda \pm i\epsilon)^{-1} \quad (8)$$

with $+$ ($-$) is the time ordered (anti time-ordered) propagator of a scalar with the squared mass equal to λ . In the framework of a time-ordered formalism $S_{\lambda}^{+}(\Phi)$ could be seen as the elementary weight.

However, with a Minkowski metric—apart from the fact that the case $\Phi = 0$ is not included in the function $w_M(\Phi^2)$ and has to be added separately—the hyperboloid pair characterised by $\Phi^2 = \text{const.}$ has infinite content. Thus with one single elementary weight the normalisation condition (2) cannot be satisfied. Further, even the difference in content between two hyperboloid pairs is in general infinite whereby a superposition of two weights does not suffice to satisfy the normalisation condition in a non-trivial way. For these reasons the minimal construction has to be:

$$w_M(\Phi^2) = \sum_{j=1}^3 a_j S_{\lambda_j}^{+}(\Phi), \quad (9)$$

with

$$\sum_{j=1}^3 a_j = 0, \quad (10)$$

and

$$\sum_{j=1}^3 a_j \lambda_j = 0. \quad (11)$$

Then the normalisation condition (2) becomes [11]:

$$\frac{4\pi^2}{4} \sum_{j=0}^3 a_j \lambda_j \ln \lambda_j = 1 - c. \quad (12)$$

The conditions (10) to (12) can be derived by putting a Fourier phase into the normalisation integral (2) and letting the variable conjugate to Φ go to zero afterwards. The conditions follow from requiring that the limit exist. Then, in general, it will also be non-zero [see Eq. (12)].

As a consequence of these conditions, w_M , as opposed to w_E , cannot be positive definite. Condition (10) resembles the one used in Pauli-Villars regularisation.

Any discrete or continuous superposition of delta weights or (time ordered) scalar propagators (8), respectively, fulfilling the normalisation condition (2) is an allowed weight function, but in what follows we will concentrate on the minimal forms given in the previous equations.

B. Generating functional

The generating functional for the time ordered Green function of QED is given by:

$$Z = Z_{int} Z_A Z_\psi, \quad (13)$$

with the interaction:

$$Z_{int} = \exp \left\{ -i \int_x \delta_\eta \not{\partial}_J \delta_\eta \right\}, \quad (14)$$

the bosonic:

$$Z_A = \exp \left\{ \frac{i}{2} \int_{x,y} J(x) \cdot \Gamma_0(x-y) \cdot J(y) \right\}, \quad (15)$$

and the fermionic part:

$$Z_\psi = \exp \left\{ -i \int_{x,y} \bar{\eta}(x) G_0(x-y) \eta(y) \right\}, \quad (16)$$

with functional derivatives δ with respect to the currents J , η , $\bar{\eta}$, and the free time-ordered propagators for the bosons Γ_0 and the fermions G_0 , respectively.

The modified theory's generating functional \mathcal{Z} is obtained by shifting the gauge field A by Φ and subsequent averaging with the weight W . Here this amounts to a modification of the fermionic part leading to:

$$\mathcal{Z} = Z_{int} Z_A Z_\psi, \quad (17)$$

where

$$\mathcal{Z}_\psi = \left\langle \exp \left\{ -i \int_{x,y} \bar{\eta}(x) G_\Phi(x-y) \eta(y) \right\} \right\rangle_W. \quad (18)$$

The other two factors of the generating functional Z_{int} and Z_A can always be taken inside the averaging integral. G_Φ is the time ordered fermion propagator in the field Φ . Under the usually made assumption [1, 2] that all other condensates are absent it obeys the equation of motion:

$$[i \not{\partial}(x) + \not{\Phi} - m] G_\Phi(x-y) = \delta^{(4)}(x-y) \quad (19)$$

which is solved by:

$$G_\Phi(z) = e^{i\Phi \cdot z} G_0(z). \quad (20)$$

Remember that the fermionic propagator in the presence of a medium resulting in a chemical potential μ reads $e^{i\mu z_0} G_0(z)$, i.e., technically the chemical potential corresponds to the temporal component of a vector and physically to a conserved charge. Carrying out the Φ integral corresponds to a Fourier transformation of the weight function:

$$\begin{aligned} \mathcal{Z}_\psi &= \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int_{\{x_m\}, \{y_m\}} \widetilde{W}(z_n) \times \\ &\times \prod_{m=0}^n [\bar{\eta}(x_m) G_0(x_m - y_m) \eta(y_m)], \end{aligned} \quad (21)$$

where $\widetilde{W}(z_n)$ is the Fourier transformation of the weight function evaluated at $z_n := \sum_{m=0}^n (x_m - y_m)$. Thence and due to the Fourier integral the prefactor can be seen as a Lorentz invariant superposition of chemical potential-like factors.

Eq. (20) has also similarities with the expressions occurring in the context of twisted boundary conditions on compact spaces which are used in lattice calculations [8]. There only the spatial components of the vector are non-zero.

The special form of the generating functional leads to the following relation for the (higher) correlators:

$$\begin{aligned} \left\langle \langle 0|T \prod_{m=1}^n \psi(x_m) \bar{\psi}(y_m) |0 \rangle \right\rangle_W &= \\ &= \widetilde{W}(z_n) \langle 0|T \prod_{m=1}^n \psi(x_m) \bar{\psi}(y_m) |0 \rangle, \end{aligned} \quad (22)$$

which remains essentially the same if bosonic operators are added. Eq. (22) evidences why \widetilde{W} has to be time ordered, if the functional is to generate time ordered Green functions.

Even if the second factor on the right-hand side of the previous equation should show gaussianity—on a given level—, i.e., factorise into two point correlators, the first factor is a genuine $2n$ -point function. Therefore the new theory is not gaussian.

Through the limitation to pure gauge configurations of the background, i.e. such with vanishing field tensor, the modified theory can be interpreted as one with a non-trivial vacuum structure without background energy

density. One could write:

$$\begin{aligned} \left\langle \left\langle 0 \left| T \prod_{m=1}^n \psi(x_m) \bar{\psi}(y_m) \right| 0 \right\rangle \right\rangle_W &=: \\ &=: \langle \Omega | T \prod_{m=1}^n \psi(x_m) \bar{\psi}(y_m) | \Omega \rangle, \end{aligned} \quad (23)$$

where $|\Omega\rangle$ stands for the new vacuum. The vacuum expectation values of the new theory $\langle \Omega | \mathcal{O} | \Omega \rangle$ are the averaged vacuum expectation values $\langle \langle 0 | \mathcal{O} | 0 \rangle \rangle_W$ of the old theory.

For two-point functions Eq. (22) together with the elementary weight (9) makes the modification of the propagator look like a contribution of a scalar to the self energy of the fermion without external fermion legs. The superposition of multiple scalars with different "mass squares" λ leads to the summation of the related self-energy bubbles. If, for example, a three-point function was constructed by including a gauge boson in the previous correlator, the modification would look like a vertex correction but still without the outer fermion legs. For correlators with more than two external fermions the correspondence to standard scalar loops does no longer persist, whence the new theory is not identical to one, where the terms for a scalar degree of freedom are added to the lagrangian density.

C. The time ordered fermion propagator

Now we study the Green function central to the modified theory, i.e., the fermionic two-point function to all orders in Φ .

1. Euclidean metric

With a euclidean metric the details of the ϵ -prescription are not important. Therefore the elementary weight of choice is the normalised delta weight $w_E^{\{\lambda\}}(\Phi^2)$ and $c = 0$. As mentioned before $\Phi^2 = 0$ in w_E also means $\Phi = 0$. However, assuming that w_E is not divergent at this point, this contribution is negligible. Denoting this special averaging procedure by $\langle \mathcal{O} \rangle_\lambda^E$ we get:

$$\langle G_\Phi(z) \rangle_\lambda^E = \frac{\sin \sqrt{\lambda z^2}}{\sqrt{\lambda z^2}} G_0(z). \quad (24)$$

This shows that, apart from an oscillatory behaviour, the propagator is suppressed over large distances $\sqrt{z^2}$. In the limit of short distances $\sqrt{z^2}$ the free propagator is recovered.

Interestingly, Eq. (24) holds not only for the free propagator $G_0(z)$ but for the full fermionic propagator, i.e., to all orders in perturbation theory. That is so, because Eq. (20) is also satisfied by the full propagator in the presence of the background Φ . Therefore, the result that

the standard propagator is recovered at small distances and that the background causes a suppression at long distances.

Back to tree-level, in momentum space we get:

$$\begin{aligned} \langle G_\Phi(k) \rangle_\lambda^E &= \frac{k+m}{4\sqrt{k^2\lambda}} \ln \left| \frac{(\sqrt{k^2} + \sqrt{\lambda})^2 - m^2}{(\sqrt{k^2} - \sqrt{\lambda})^2 - m^2} \right| + \\ &+ \frac{k}{4k^2} \left[2 - \frac{k^2 + \lambda - m^2}{2\sqrt{k^2\lambda}} \ln \left| \frac{(\sqrt{k^2} + \sqrt{\lambda})^2 - m^2}{(\sqrt{k^2} - \sqrt{\lambda})^2 - m^2} \right| \right] \end{aligned} \quad (25)$$

One can see that the on-shell pole has been removed from the propagator. It has been replaced by one proportional to $1/\sqrt{k^2}$. Consequently the elementary fermions have been removed from the spectrum of asymptotic states.

This result is similar to the one in [1, 2], but where also background configurations with non-vanishing field tensors were admitted in addition to the pure gauge configurations used here exclusively. In coordinate space the weight chosen in [1] constrained to pure gauge backgrounds yields :

$$\langle G_\Phi(z) \rangle_{HP}^E = \exp(-z^2 \Lambda^2/4) G_0(z) \quad (26)$$

Here as well the free propagator is recovered at small z^2 and damped at large z^2 . Even taking the average with this special weight for $2n$ -point fermion correlators does not lead to a factorisation into two-point correlators (gaussianity).

2. Minkowski metric

In Minkowski space, if one wants to stick to a time ordered treatment the adapted weight function has to be chosen for the additional contribution. In coordinate space, taking the weight given by Eq. (3) with Eq. (9) and $c = 0$ leads to:

$$\langle G_\Phi(z) \rangle_M^{\{\lambda_j\}} = 4\pi^2 \sum_{j=1}^3 a_j \sqrt{\lambda_j} \frac{K_1(\sqrt{\lambda_j} \sqrt{-z^2 + i\epsilon})}{\sqrt{-z^2 + i\epsilon}} G_0(z) \quad (27)$$

At $z^2 = 0$ the prefactor of the free propagator $G_0(z)$ in the previous equation goes to 1 due to Eqs. (10) to (12) with $c = 0$. Thence, the free propagator is recovered in the limit of small z^2 . If c is chosen different from zero, once the free propagator is still reproduced taken together with the explicitly free contribution. The reproduction of the free propagator at $z^2 = 0$ is an intrinsic consequence of the need to normalise.

Like in euclidean space the previous relation also holds for the full propagator, for the same reason as there.

If $\lambda_j > 0 \forall j \in \{1; 2; 3\}$, for $z^2 \rightarrow +\infty$ the envelope of this function decays proportionally to $(z^2)^{(-3/4)}$; for $z^2 \rightarrow -\infty$ proportionally to $(-z^2)^{(-3/4)} \exp[-\sqrt{-\min(\{|\lambda_j|\})z^2}]$. If $\lambda_j < 0 \forall j \in \{1, 2, 3\}$ the two cases are exchanged.

Thus, for large absolute values of z^2 , $\langle G_\Phi(z) \rangle_M^{\{\lambda_j\}}$ is suppressed relative to a free propagator, which shows that the fermions cannot propagate over arbitrarily large distances.

In momentum space the form of $\langle G_\Phi(k) \rangle_M^{\{\lambda_j\}}$ can be determined best by making use of its correspondence to the one-loop contribution of a scalar to the self-energy of the fermion. One obtains:

$$\begin{aligned} \langle G_\Phi(k) \rangle_M^{\{\lambda_j\}} &= \\ &= i\pi^2 \sum_{j=0}^3 a_j \int_0^1 dx (x \not{k} + m) \ln |(x - x_j^+)(x - x_j^-)| \end{aligned} \quad (28)$$

with

$$x_j^\pm = \frac{\lambda_j + k^2 - m^2}{2k^2} \pm \sqrt{\left(\frac{\lambda_j + k^2 - m^2}{2k^2}\right)^2 + \frac{\lambda_j}{k^2}}, \quad (29)$$

$\forall j \in \{1; 2; 3\}$. For $\lambda_j > 0 \forall j \in \{1; 2; 3\}$ the x -integration can be carried out yielding:

$$\begin{aligned} \int_0^1 dx \ln |x - x_j^\pm| &= \\ &= (1 - x_j^\pm) \ln |1 - x_j^\pm| + x_j^\pm \ln |x_j^\pm| - 1 \end{aligned} \quad (30)$$

and

$$\begin{aligned} \int_0^1 x dx \ln |x - x_j^\pm| &= \\ &= \frac{1 - (x_j^\pm)^2}{2} \ln |1 - x_j^\pm| + \frac{(x_j^\pm)^2}{2} \ln |x_j^\pm| - \frac{x_j^\pm}{2} - \frac{1}{4}. \end{aligned} \quad (31)$$

$\langle G_\Phi(k) \rangle_M^{\{\lambda_j\}}$ is free of poles. For small k^2 and small mass m^2 , the propagator becomes:

$$\langle G_\Phi(k) \rangle_M^{\{\lambda_j\}} \approx i\pi^2 (\not{k}/2 + m) \sum_{j=1}^3 a_j \ln |\lambda_j|. \quad (32)$$

For large k^2 , $\langle G_\Phi(k) \rangle_M^{\{\lambda_j\}}$ becomes proportional to k^{-2} reproducing the behaviour of the free propagator.

Independent of the details, for $c = 0$ no freely propagating particles are described by the propagator in an ensemble of pure gauge backgrounds. This also explains its aforementioned correspondence to self-energy contributions from scalars *without* external fermion legs.

III. SUMMARY

We have studied gauge field theories with restored Lorentz invariance. Starting out with a manifestly Lorentz invariant field theory, this symmetry is broken through the inclusion of a (non-trivial) dependence on a four-vector Φ . In the explicitly investigated examples said vector plays the rôle of a contribution to the gauge field. The symmetry is restored by defining correlators as average over a Lorentz invariant ensemble of

vectors. Apart from the original contribution with $\Phi = 0$ the additional term is characterised by a weight function of the only Lorentz invariant Φ^2 . Therefore these theories are connected to mass dimension two vector condensates. The modifications can also be interpreted as a means to include the effect of a non-trivial Lorentz invariant vacuum structure into the original theory. Some of the structures also appear in stochastic field theories. The resulting theories do not show gaussianity.

The presence of the background can bar the asymptotically free propagation of the matter fields used to write down the lagrangian density, in euclidean and Minkowski space. The bare propagator without the background is still reproduced at short distances and high momenta. The dressed propagator has no on-shell pole and is suppressed at large distances relative to the undressed one. In non-abelian gauge theories also a constant gauge field can contribute to the field tensor. Commonly, within this setting, ensembles of backgrounds are used which contain pure gauge configurations and field configurations leading to a non-zero field tensor. Therefore it was not clear *a priori* to which contributions the observed effect is connected. We find that when constraining the ensemble to pure gauge configurations, the effect is still present. That the pure gauge configurations do lead to non-trivial phenomena can be understood from the mentioned analogy to a chemical potential. Remarkably we have been able to show the suppression of the long-range propagation not only based on the fermionic propagator to all orders in the background and otherwise at tree level but for the full fermionic propagator, i.e., to all loops.

For correlators involving two fermion fields in general and thus especially for the fermion propagator the modifications due to the background resemble scalar loops bridging the fermion lines. The corresponding diagrams do not carry external fermion legs although they are direct contributions to the propagator indicating in this way that they do not involve freely propagating particles.

Some extensions of the standard model violating Lorentz invariance are based on the concept of non-commutative field theories. As here Lorentz invariance has been restored, it would be interesting to study the relationship between the present approach and non-commutative field theories not breaking Lorentz invariance [9].

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